**1. Log-Linear Model**

In the log-linear model, the dependent variable *y* is log-transformed, whereas the predictor variable *x* is not. The regression equation is:

But how do we interpret the results of the log-linear model?

Here, is the expected change in ln(y) when *x* goes up by 1 unit. Said differently, as *x* goes up by 1 unit, the expected value of *y* goes up by .

Let’s imagine we have 2 values of *y* – that is, and :

**(2)**

**(3)**

Let’s subtract the equation (2) from equation (3) to examine what happens when *x* goes up by 1 unit:

**(4)**

Because , equation **(4)** may be rewritten as:

**(5)**

Let’s exponentiate both sides of equation **(5)**:

**(6)**

Let’s add and subtract 1 from the right-hand side of the equation. Adding and then subtracting 1 doesn’t change the original right-hand side of the equation.

**(7)**

From **(6)** and **(7)**, it follows that:

**(8)**

Now, let’s subtract 1 from both sides of equation **(8)**, and multiply both sides by 100%.

**(9)**

Now, let’s recognize that the right-hand side of equation **(9)** is simply the % change in *y*. That is, the % change in *y* is calculated as .

We can interpret all of this as follows: as *x* goes up by 1 unit, the expected change in our dependent variable *y* is .

However, when is small (≤ 0.3), then

**(10)** 100%,

so we can say that as *x* goes up by 1 unit, the expected change in our dependent variable *y* is approximately 100%.

The table below shows that this approximation is no longer accurate when is big. If is big (> 0.3), do not use the approximation in **(10)**.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | **Approximation: 100%** | **Verdict** |
| 0.001 | 0.1001 | 0.1 | Very close - great approximation |
| 0.01 | 1.005 | 1 |
| 0.05 | 5.1271 | 5 |
| 0.1 | 10.5171 | 10 |
| 0.15 | 16.1834 | 15 | Somewhat close – relatively decent approximation |
| 0.2 | 22.1403 | 20 |
| 0.25 | 28.4025 | 25 |
| 0.3 | 34.9859 | 30 |
| 0.35 | 41.9068 | 35 | Not very close - bad approximation |
| 0.4 | 49.1825 | 40 |
| 0.5 | 64.8721 | 50 |
| 0.8 | 122.5541 | 80 |
| 1 | 171.8282 | 100 |
| 1.5 | 348.1689 | 150 |
| 2 | 638.9056 | 200 |
| 5 | 14741.3159 | 500 |
| 10 | 2202546.58 | 1000 |

Also of interest is what happens when is negative. In these instances, the approximation works great when > -0.1, and relatively well when . It doesn’t work well when < -0.2.

**2. Linear-Log Model**

In the linear-log model, the predictor variable *x* is log-transformed, whereas the dependent variable *y* is not. The regression equation is:

**(11)**

But how do we interpret the results of the linear-log model?

Let’s imagine we have 2 values of *y* – that is, and :

**(12)**

**(13)**

That is, in **(13)** is the expected value of *y* when goes up by 1 *unit*.

Now, let’s recall the natural number, *e* 2.72. Remembering that , meaning that we can rewrite **(13)** as:

**(14)**

Since the sum of the natural logarithms of two numbers is the natural logarithm of the product of these numbers, from **(14)** it follows that:

**(15)**

Equation **(15)** implies that:

* As goes up by 1 unit, the expected change in *y* is units.
* Adding 1 to is the same as multiplying *x* by approximately 2.72. So as we multiply *x* by 2.72, the expected change in *y* is units.
* Multiplying *x* by 2.72 is the same as increasing *x* by 172% = 100 \* (2.72 – 1)%. So as we increase *x* by 172%, the expected change in *y* is units.

But what happens to *y* if *x* goes up by 1%? Note that here, we’re looking at a 1% increase in *x*, and not . We can look at equation **(16)** below, where we see a 1% increase in *x* (from **(12)** above).

**(16)**

If we subtract equation **(12)** from **(16)**, we get:

**(17)**

And **(17)** implies that as goes up by 1%, the expected change in y is units.

However, because , we have the following approximation, which works for any value of .

**(18)**  .

**3. Log-Log Model**

In the log-log model, both the dependent variable *y* and predictor *x* are log-transformed. The regression equation is:

**(19)**

We can say that multiplying *x* by *e* = 2.72 will multiply the expected value of *y* by .

To have a better understanding of this interpretation, let’s imagine that we have 2 values of *y* – that is, and , and that:

**(20)**

**(21)**

Let’s subtract **(20)** from **(21)**:

**(22)**

Simplifying **(22)**, we get:

**(23)**

.

Using the same logic as in equation **(7)**, we can rewrite the left-hand side of equation **(23)** as follows:

**(24)**

So, from **(23)** and **(24)**, it follows that:

**(25)** .

Now, recall from the properties of logarithms that

**(26)** .

That is, equation **(25)** can be rewritten as:

**(27)** .

Exponentiating both sides of **(27)** yields:

**(28)** .

We can rearrange **(28)** and multiply both sides by 100% to get:

**(29)** .

Again, we can recognize that the left-hand side of equation **(29)** is simply the % change in *y*. We can interpret the equation as follows: as *x* changes by 1%, the expected value of *y* changes by .

However, when is small (< 20 or 30), we can use the following approximation:

**(30)**

The table below shows how well the approximation works for different values of :

|  |  |  |
| --- | --- | --- |
|  |  | **Verdict** |
| 0.01 | 0.010 | Great approximation |
| 0.1 | 0.100 |
| 0.5 | 0.499 |
| 1 | 1.000 |
| 2 | 2.010 |
| 5 | 5.101 |
| 8 | 8.286 |
| 10 | 10.462 |
| 15 | 16.097 | Decent approximation |
| 20 | 22.019 |
| 30 | 34.785 |
| 50 | 64.463 | Poor approximation |
| 100 | 170.481 |
| 500 | 14377.277 |

If is large and the approximation in **(30)** doesn’t work well, then simply use the interpretation given after equation **(29)**: as *x* changes by 1%, the expected value of y changes by .

Also of interest is what happens when is negative. In these instances, the approximation works great when > -10, and relatively well when . It doesn’t work well when < -20.